RADIAL ELECTRIC FIELDS EXCITED IN AIR BY A POINT SOURCE OF γ-QUANTA

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INTRODUCTION

A whole series of papers [1-6] has been devoted to the investigation of the characteristics of a radial electric field resulting from the polarization of air by the flow of nonsteady currents of Compton electrons excited by a pulse of γ-quanta. In the majority of these papers (see, for example, [1-3]) the derivation of the formulas for the current density of the Compton electrons did not take into account their deceleration in the resulting electric fields, which is valid at quite low altitudes, where the air density is close to normal. In [4], another limiting case is considered — the region of low air densities. In this region, deceleration of the Compton electrons due to collisions with air molecules can be neglected, and when deriving the formula for the current density, only the deceleration of the electrons by the electric field need be taken into consideration. A simultaneous calculation of the effect of these two factors on the motion of the Compton electrons was made in [5] for the case of slowly varying fields and functions describing the time-dependence of the γ-quanta yield on the source, while in [6] the case of a short [δ(t)] pulse of γ-quanta was considered. Based on the solution of the model kinetic equation, below we obtain a formula for the current density of the Compton electrons, taking into account the effect of the electric field and ionization deceleration on the motion of the electrons over a quite wide range of change of air density and with arbitrary time-dependences of the γ-quanta yield from the source.

It follows from [4] that with low air densities, the electric fields attain values which can increase the energy of the conductivity electrons up to quite high values. This leads, on the one hand, to a decrease of the electron mobility and, on the other hand, to additional ionization of the air by these electrons. Thus, a correct calculation of the dependence of the electron drift and the coefficient of ionization on the magnitude of the electric field becomes essential.

This paper is devoted to the consideration of these problems.

§1. When deriving the formula for the current density of Compton electrons, it is assumed, as in [1-6], that all electrons are created with identical initial momentum $P_0 = \sqrt{W_0(W_0 + 2mc^2)/c^2}$ ($W_0 \sim 1$ MeV is the initial kinetic energy of the electron) directed along a radius. Therefore, the number of Compton electrons originating in unit time in unit volume of phase space (r, P) at a distance r from the source is $S(r, P, \tau) = n_k\delta(P - P_0)g(\tau)$, where $n_k = Ne^{-r/\lambda}/4\pi r^2\lambda$ is the concentration of Compton electrons at the point being considered, originating during all the time of operation of the source; N is the total number of γ-quanta; λ is the range of the γ-quanta; g(τ) is a function defining the change of γ-flux with time $\left(\int\limits_0^\infty g(\tau)\,d\tau = 1\right)$; and $\tau = t - r/c$ is the time, read from the instant of arrival of the γ-quanta at the point being considered.

The change of energy of an electron because of collisions with air molecules is described in the approximation of continuous moderation [7], according to which a retarding force F acts on the electron. In the range of electron energies being considered (~1 MeV), we can put $F = W_0/l$ = const (l is the range of a Compton electron). Just as in [5, 6], the effects due to multiple elastic scattering of the electrons [8] are not taken into consideration.

It is assumed also that the drift of the Compton electrons is considerably less than their Larmor radius in the geomagnetic field $B_0 = 0.5$ Oe. In this case, it can be assumed that the radial velocity of the Compton electrons changes little due to twisting in the

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magnetic field, and, therefore, the radial motion of the electrons is determined only by the ionization losses and by deceleration by the polarized electric field E, directed along the radius. The limits of applicability of this assumption, depending on the local characteristics of the γ-flux, will be determined below.

In the future, we shall consider distances from the source of the γ-quanta at which the drifts of the Compton electrons are small in comparison with these distances. This permits us to assume that all functions are dependent only on τ [4].

Within the scope of the assumptions made, the kinetic equation for the distribution function of the Compton electrons has the form

$$\left(1-\frac{v}{c}\right)\frac{\partial f}{\partial \tau}-[eE(\tau)+F]\frac{\partial f}{\partial P}=S(P,\ \tau). \tag{1.1}$$

Integrating Eq. (1.1) along the characteristic curve, the current density of the Compton electrons can be expressed in terms of the "potential" $\Phi(\tau)=\int\limits_0^\tau E(\tau')\,d\tau'$ of the electric field E(τ):

$$j(\Phi(\tau),\ \tau)=-en_k\int\limits^{\tau} d\tau' g(\tau')\frac{v(\tau,\ \tau')}{1-v(\tau,\ \tau')/c}, \tag{1.2}$$

where

$$v(\tau,\ \tau')=c\,\frac{1-[\beta+(e\Phi(\tau)-e\Phi(\tau'))/mc+F(\tau-\tau')/mc]^2}{1+[\beta+(e\Phi(\tau)-e\Phi(\tau'))/mc+F(\tau-\tau')/mc]^2}$$

is the velocity of an electron, created at the instant τ', at the instant τ ($\beta=\sqrt{1+(P_0^2/m^2c^2)}-(P_0/mc)$; m and e are the mass and charge of the electron; and c is the velocity of light).

The integration in Eq. (1.2) is carried out over the region $v(\tau,\ \tau')\geqslant 0$, i.e., it is assumed that an electron moderated to energies of $W \ll mc^2$ is "absorbed" by the medium. A more detailed description of the motion of the electrons in the low-energy region leads to small corrections in Eq. (1.2) [9]. Within the scope of the model being considered, the "lifetime" of a Compton electron $\theta(\tau,\ \tau_*)$ created at the instant τ can be introduced [τ_* is determined from the condition $v(\tau_*,\ \tau)=0$].

For different limiting cases, expressions for the current density of Compton electrons which have been used by the authors of a number of papers [1-6] can be obtained from Eq. (1.2):

1. If $eE/F \ll 1$, then the effect of the resulting electric field on the motion of the electrons can be neglected. In this case

$$j=-en_kc\int\limits^{\tau} d\tau' g(\tau')\frac{1-[\beta+F(\tau-\tau')/mc]^2}{2[\beta+F(\tau-\tau')/mc]^2}. \tag{1.3}$$

This expression for the current density of Compton electrons was obtained in [3].

2. For slowly varying source functions [characteristic time of variation g(τ) is considerably less than $\theta=mc(1-\beta)/F$], taking out g(τ) from under the integral sign in Eq. (1.3), the expression

$$j=-en_klg(\tau) \tag{1.4}$$

can be obtained; this expression coincides with that used in [1, 2].

3. If E(τ) and g(τ) are varying weakly during the "lifetime" of the Compton electron, then expanding Φ(τ') in formula (1.2) up to linear terms in (τ − τ'), and taking out g(τ) from under the integral sign, we arrive at the expression

$$j=-en_kl\frac{g(\tau)}{1+\frac{eEl}{W_0}}, \tag{1.5}$$

which was used in [5].

4. In a sufficiently rarefied atmosphere, where collisions of Compton electrons with the air molecules can be neglected, we obtain [4]

$$j = -en_k \frac{c}{2} \int_0^\tau d\tau' g(\tau') \frac{m^2c^2 - \left[\sqrt{P_0^2 + m^2c^2} - P_0 + e\Phi(\tau) - e\Phi(\tau')\right]^2}{\left[\sqrt{P_0^2 + m^2c^2} - P_0 + e\Phi(\tau) - e\Phi(\tau')\right]^2}. \tag{1.6}$$

Thus, the formula (1.2) derived for the current density of Compton electrons is obviously quite universal. For different limiting cases, it correctly reproduces the behavior of the current and converts to the well-known formulas obtained previously from other considerations and by other methods.

§2. The limits of applicability of formulas (1.3)-(1.6) are determined by the characteristics of the source and the radial electric field, which, in turn, depend in many respects on the resulting nonsteady conductivity of the medium. As already mentioned above, in a sufficiently rarefied atmosphere, the development of electron showers may occur, these showers originating due to ionization of the air by conductivity electrons accelerated by the electric field. In [4], the ionization frequency $\nu_i(T_e)$ was determined by the temperature of the conductivity electrons T_e, which was found from the energy balance, taking into consideration the Joule heating and loss of energy of the electrons due to ionization. It was shown that for values of the dimensionless parameter $A = [(2W_0/I)(\omega^2_p/\nu_{st}\nu_i(I))]^{1/2} > 10$ (I is the ionization potential of air, ν_{st} is the elastic collision frequency of the conductivity electron with the air molecules, and $\omega^2_p = 4\pi e^2 n_k/m$); the temperature of the electrons in the region of large values of the field ($E \sim \sqrt{8\pi n_k W_0}$) is greater than or is of the order of the ionization potential I of air. Consequently, for those values of the parameter of the problem at which $A \gg 1$, inelastic collisions play the principal role in the energy balance of the conductivity electrons. Therefore, in the region where effectiveionization is achieved, the distribution function of the conductivity electrons is finely tuned below the local value of the electric field, and the ionization frequency is a function of the field E [10].

In this paper, for the purpose of a more correct computation of the effect of conductivity on the characteristics of the electric field, empirical dependences of the ionization frequency $\nu_i(E) = \alpha_T(E/p)pv_e(E/p)$ and the drift velocity $v_e(E/p)$ of the conductivity electrons on E/p were used (p is the air pressure).

Averaged over the result of various experimental papers [11] in the range $0 \leqslant E/p \leqslant 1000$ V/cm•mm Hg, these relations have the following form for the electron drift velocity v_e:

$$v_e = \begin{cases} 6.33\cdot 10^6\ E/p, & 0 \leqslant E/p \leqslant 0.036, \\ 1.20\cdot 10^6 \sqrt{E/p}, & 0.036 \leqslant E/p \leqslant 3.46, \\ 8.80\cdot 10^5 (E/p)^{3/4}, & 3.46 \leqslant E/p \leqslant 175, \\ 3.20\cdot 10^6 \sqrt{E/p}, & 175 \leqslant E/p \leqslant 1000; \end{cases} \tag{2.1}$$

for the Townsend ionization coefficient α_T;

$$\alpha_T = \begin{cases} 2.57\cdot 10^{-8} \exp(0.352\ E/p), & 0 \leqslant E/p \leqslant 36, \\ 1.21\cdot 10^{-4} (E/p - 27.8)^2, & 36 \leqslant E/p \leqslant 130, \\ 0.511\cdot \sqrt{E/p} - 4.57, & 130 \leqslant E/p \leqslant 1000 \end{cases} \tag{2.2}$$

(v_e, cm•sec^{-1}; α_T, cm^{-1}•mm Hg).

The decrease of the concentration of conductivity electrons as a result of adhesion to air molecules is not given consideration in this paper, since in a rarefied atmosphere $\delta < 10^{-1}$ ($\delta = p/p_0$ is the ratio of the air pressure to the normal pressure), the characteristic adhesion time is $\gamma^{-1} \gtrsim 10^{-7}$ sec [11]. Therefore, for sources with a characteristic time of output of γ-quanta $\tau_0 \lesssim 10^{-7}$ sec (which are also examined in this paper), this process cannot be considered.

The change of concentration of the conductivity electrons due to electron—ion recombination also is not taken into consideration since, as was shown in [6], the characteristic time of this process is considerably greater than the attenuation time of the field due to conductivity, even without allowing for the formation of a shower.

Thus, the system of equations describing the change of the electric field and the concentration of conductivity electrons n_e with time acquires the form

$$d^2\Phi/d\tau^2 = -4\pi\{j + en_e v_e(E/p)\},$$
$$dn_e/d\tau = \nu|j|/el + \alpha_T(E/p)pv_e(E/p), \qquad (2.3)$$

where $\nu = 3\cdot10^4$ is the number of electron—ion pairs formed as a result of the absorption of 1 MeV of energy in air, and $v_e(E/p)$ and $\alpha_T(E/p)$ are determined by formulas (2.1) and (2.2).

§3. If the dimensionless variables $t = \tau/\tau_0$, $y = (E/E_0)$ $(E_0 = \sqrt{8\pi n_k W_0})$, and $z = n_e/\nu n_k$ are introduced, then the system of equations (2.3) will depend on two dimensionless parameters: $\alpha = eE_0 l/W_0$ and $\xi = \tau_0\omega_p$. The first of these defines, just as in [4], the relative magnitude of the electric field, while the second defines the width of the γ-pulse. For $\xi \ll 1$, the results should depend weakly on the form of the function describing the time dependence of the γ-quanta yield on the source; in order to estimate the amplitude values of the fields E_m (for $\alpha \gtrsim 1$), the formula $E_m \simeq E_0$ obtained in [4] for a $\delta(\tau)$-source is used. For $\xi \gtrsim 1$, the results will have a less universal nature and will be determined, to a large degree, by the form of $g(\tau)$.

It has not been the aim in this paper to investigate in detail the dependence of the results on the source function; therefore, only one form of the time dependence of $g(\tau)$ was used for the calculations:

$$g(\tau) = \frac{\tau}{\tau_0^2}\exp\left(-\frac{\tau}{\tau_0}\right),\ \tau_0 = 10^{-8}\ \text{sec.}$$

The system of equations (2.3), with the expression for the current in the form of Eq. (1.2), was solved numerically for the following values of the parameters: $N = 10^{23}$, $\lambda_0 = 2.5\cdot10^4$ cm $(\lambda = \lambda_0/\delta)$, $l_0 = 3\cdot10^2$ cm $(l = l_0/\delta)$, and $\delta = 10^{-1}$-10^{-3}.

Figure 1 shows the dependence of the electric field (in units of E_0) on the time for three values of δ and for a distance from the source of $r = 10^5$ cm. The results (dashed curve) obtained without allowing for the development of a shower in the electric fields $(\nu_i \equiv 0)$ are also represented here.

It can be seen that due to the development of a shower, the time characteristics of the field pulse are reduced the stronger, the more rarefied the air. This dependence on δ can be understood qualitatively if we accept the breakdown model [12]. Within the scope of this model, the condition of development of a shower in the quasisteady case has the form

$$\nu_E = e^2E^2/m\nu_{st}I > \nu_*, \qquad (3.1)$$

where ν_* is the characteristic frequency of inelastic collisions of the conductivity electrons with the air molecules; ν_E is a quantity, the reciprocal of the time of increase of the electron energy by the action of the field up to a value J, which is sufficient for multiplication. Since the maximum value of the electric field in the problem being considered is estimated by the formula $E_m \simeq E_0 (\xi < 1)$, condition (3.1) is equivalent to fulfillment of the inequality

$$\eta = \sqrt{\frac{2W_0}{I}\frac{\omega_p^2}{\nu_{st}\nu_*}} > 1. \qquad (3.2)$$

This condition coincides with that obtained in [4], if we put $\nu_* = \nu_i(I)$. It follows from Eq. (3.2) that $\eta \sim 1/\sqrt{\delta}$, and, therefore, with a decrease of the air density, a shower develops in an even wider range of values of the electric field. Substituting the numerical values of the parameters from [13], it can be verified that the condition for the origination of a shower — Eq. (3.2) — and the condition

$$\alpha = eE_0 l/W_0 > 1 \qquad (3.3)$$

coincide:

$$\eta/\alpha = W_0/l\sqrt{mI\nu_{st}\nu_*} \simeq 1.$$

Therefore, if $\alpha \gg 1$, it is necessary to take into consideration both the effect of the electric field on the motion of the Compton electron and the development of an electron shower in the resulting fields. Condition (3.3) is satisfied, if the flux of γ-quanta at the point being considered is

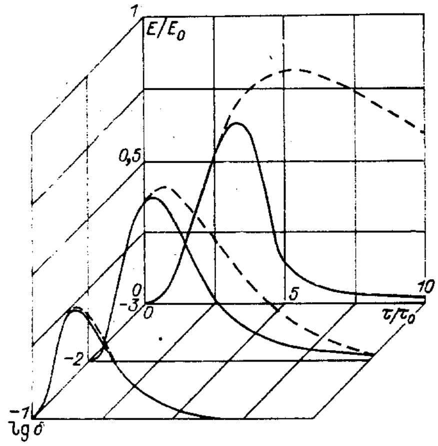

Fig. 1

$$I_\gamma = \frac{N\,e^{-r/\lambda}}{4\pi r^2} \gg I_1 \simeq 0.8 \cdot 10^{11}\,\delta. \tag{3.4}$$

In deriving formula (1.2), the assumption was used about the smallness of the drift of the Compton electrons $c\theta$ in comparison with their gyromagnetic radius c/ω_H. Since with a decrease of δ the drift of the Compton electrons increases, in order to estimate the limits of applicability of the model, the formulas for $\alpha \gg 1$ can be used. In strong electric fields, the characteristic lifetime of the electrons is

$$\theta \simeq \frac{(1-\beta)\,mc^2}{W_0}\,\frac{l}{c\,(1+\alpha)} \simeq \frac{0.4}{\omega_p}.$$

Therefore twisting of the Compton electrons in the magnetic field can be neglected, if $\omega_H/\omega_p < 5/2$, whence the condition of applicability of formula (1.2) is obtained:

$$I_\gamma > I_2 \simeq 10^8 \delta^{-1}. \tag{3.5}$$

It should be noted that for the values of the parameters given above ($N = 10^{23}$ and $r = 10^5$ cm), the flux of γ-quanta $I_\gamma = 0.8 \cdot 10^{12}$ quanta/cm^2, and conditions (3.4) and (3.5) are satisfied in the range of values of $\delta = 10^{-3}$–10^{-1}, which also were used for the calculations.

It will be interesting to determine the limits of applicability of the simpler formulas (1.3)-(1.5) for the current density of Compton electrons. For $I_\gamma < I_1$, the effect of the resulting field on the motion of the electrons can be neglected, and instead of Eq. (1.2), formula (1.3) can be used. If, in this case, the condition $\tau_0 \gg \theta \approx 0.4l/c = 0.4 \cdot 10^{-8} \delta^{-1}$ is also satisfied, then in place of Eq. (1.3), the simpler formula (1.4) can be used. For $I_\gamma < I_1$, no electron shower develops in the resulting field.

In the most interesting range of values of the parameters $I_\gamma > I_1$, in which the effect of the electric field on the motion of the Compton electrons and the conductivity electrons is considerable, formula (1.5) can be used. It should be noted, however, that its limits of applicability are confined to the condition $\tau_0 \gg \theta \approx 0.4/\omega_p \sim 1/\sqrt{\delta}$. Therefore, with increase of altitude, this condition can be violated. In particular, with the chosen values of the parameters, this occurs for $\delta \approx 10^{-2}$. With smaller values of δ, it is necessary to use formula (1.2).

LITERATURE CITED

1. A. S. Kompaneets, "Radio emission of a nuclear explosion," Zh. Éksp. Teor. Fiz., 35, No. 6(12), 1538 (1958).
2. V. Gilinsky "Kompaneets model for radio emission from a nuclear explosion," Phys. Rev., 137, No. 1A, 50-55 (1965).
3. W. Y. Karzas and R. Latter, "Detection of the electromagnetic radiation from nuclear explosions in space," Phys. Rev., 137, No. 5B, 1369-1378 (1965).
4. M. F. Ivanov, A. A. Solov'ev, and V. A. Terekhin, "Self-consistent problem of electric fields created in the air by a pulse of γ-quanta," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 7-10 (1975).
5. Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "The electric field created in air by a pulse of γ-quanta," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 3-8 (1970).

6. Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "Pulsed function of streams of Compton electrons," in: Problems of the Metrology of Ionizing Radiations [in Russian], Atomizdat, Moscow (1976).
7. O. F. Evdokimov, "Multiple scattering of fast electrons in a gas in the presence of an electric field," Zh. Tekh. Fiz., 45, No. 3, 593-599 (1975).
8. A. V. Zhemerev, Yu. A. Medvedev, and B. M. Stepanov, "Pulsed current of electrons excited by γ-radiation in air," Atom. Energ., 41, No. 4, 268-269 (1976).
9. A. V. Grevich, "Theory of the effect of escaping electrons," Zh. Eksp. Teor. Fiz., 39, No. 5(11), 1296-1307 (1960).
10. S. C. Brown, Introduction to Electrical Discharges in Gases, Wiley (1966).
11. J. Dutton, "A survey of electron-swarm data," J. Phys. Chem. Ref. Data, 4, No. 3, 577-856 (1975).
12. Yu. P. Raizer, Laser Flash and the Propagation of Discharges [in Russian], Nauka, Moscow (1974).
13. L. E. Kline and Y. G. Siambis, "Computer simulation of electrical breakdown in gases: avalanche and streamer formation," Phys. Rev., 5, No. 2A, 794-805 (1972).

IONIZATION AND RECOMBINATION OF A MULTICHARGED PLASMA HEATED BY LASER RADIATION

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Ionization and recombination processes taking place in a laser plasma during its heating up and subsequent cooling play an important role in the formation of the charge and energy spectra of the plasma ions [1, 2]. It appears that recombination during dispersion of the plasma does not lead to the total disappearance of the charged particles. The possibility of hardening the degree of ionization was shown theoretically for the first time in [3]. A subsequent study of the dispersion of a previously heated and ionized plasma [4-6] showed that the most effective hardening occurs at the periphery of the plasma bunch, i.e., where the rate of expansion of the plasma is the highest and the density is the lowest. However, in all these papers, the stage of heating up of the plasma by laser radiation has not been considered.

This paper is devoted to the numerical investigation of the dispersion of a multicharged plasma (aluminum and deuterium–carbon), subjected to the action of the radiation of a neodymium laser. The following inelastic processes are taken into consideration for the calculations: ionization by electron shock, photorecombination, ternary recombination at the ground level with an electron as the third particle, stopping absorption of radiation incident on the plasma, bremsstrahlung, electron thermal conductivity, and energy transfer between electrons and ions.

The dispersion of a continuous spherical bunch of multicharged plasma, in which the inelastic processes mentioned above occur, can be considered by means of the equations of gas dynamics, using the assumption of quasineutrality of the plasma. Breakdown of quasineutrality occurs at distances on the order of the Debye radius. For the plasma parameters used (density $\sim 10^{22}$ cm^{-3}, temperature $\sim 10^{2}$–10^{3} eV), the Debye radius $r_D \sim 10^{-6}$ cm, which is considerably less than the characteristic size of the plasma bunch $R \sim 10^{-2}$ cm. Therefore, breakdown of quasineutrality can occur only close to the edge of the plasma, and at quite large dimensions of the bunch, the effect of the resulting electrical forces on the motion of the main bulk of the plasma is negligibly small [7]. The effect of the electric field can result, however, at distances of the mean free path of ions relative to ion–ion collisions l_{ii}. In fact, the self-consistent electric field originating even under the condition of quasineutrality leads to ions of different charges being accelerated differently in this field. At the same time ion–ion collisions create a strong friction which balances the velocities of the ordered motion of ions with different charges. This velocity balancing takes place over the whole volume of the plasma, with the exception of a small region of the order of l_{ii}, immediately

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